2018 Fall EECS205003 Linear Algebra - Midterm

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1. (13%) Consider two vectors u = 1

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 *∈ R*3 and v =

0 1 1

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 *∈ R*3.

(a) (3%) Find *k*u*k*, *k*v*k* and the angle *θ* between u and v.

(b) (3%) Find the perpendicular projection of v on u.

(Hint: *<*v*,*u*>*

*k*u*k*~~2~~ u)

(c) (4%) Find a vector w *∈ R*3such that w is perpendicular to both u and v. (d) (3%) Consider the space *P*2 with inner product:

*< p, q >*= *p*(0)*q*(0) + *p*(12)*q*(12) + *p*(1)*q*(1).

Is *p*(*x*) = 4*x*2 *−* 4*x* + 1 orthogonal to *q*(*x*) = 4*x*2 *−* 1? Please give your reason. (Hint: *Pn ≡ {f*(*x*)*|f*(*x*) = *anxn* + *an−*1*xn−*1 + *· · ·* + *a*1*x* + *a*0*, ai ∈ R}*)

2. (10%) For *A*x = b where *A* =

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2 8 7 *−*1 *−*4 6 3 12 7

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*b*1

*b*2

*b*3

*b*4

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(a) (3%) Describe the column space C(*A*) of *A*. What is the rank of *A*?

(b) (4%) For which b (find a condition on *b*1*, b*2*, b*3*, b*4) is *A*x = b solvable?

(c) (3%) If we remove a column from *A* and *A*x = b is still solvable, which column would it be? Please give your reason.

3. (12%) Test if the following sets of vectors form subspaces .

(a) (2%) *{* (*x*1*, x*2*, x*3) *∈ R*3: *x*1 + 2*x*2 + 3*x*3 = 0 *}*

(b) (2%) *{* (*x*1*, x*2*, x*3) *∈ R*3: *x*1 + 2*x*2 + 3*x*3 = 4 *}*

(c) (2%) The set of all symmetric matrix.

(d) (2%) The set of all nonsingular matrix.

(e) (2%) *{* (f) (2%) *{*

*a* 0 *b c**a* 0 *b c*

*∈ M | a* + *b* + *c* = 0 *}*

*∈ M| ∀ a, b, c ∈ R }*

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4. (15%) Let matrix *A* =

1 2 1 3 7 6 1 4 8

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(a) (4%) Find the LU factorization of A. Use the answer to solve Ax = *−*12

(b) (4%) Use Gauss-Jordan Elimination method to find *L−*1 and *U−*1. Use the answers to find *A−*1.

Please turn over to continue.

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(c) (3%) Consider the following linear equations, solve for x and y by using elimination and back substitution.

2*x* + 3*y* = 5

6*x* + 15*y* = 12

(d) (2%) Prove for any permutation matrix *P*, *P P T* = *I*.

(e) (2%) Find 3 *×* 3 permutation matrices with *P*3 = *I* (but not *P* = *I*).

5. (18%) Consider the system of linear equations:

*x − y* + 4*z* = *−*11

*x −* 2*y* + 3*z* = *−*13

*−*2*x* + *y −* 3*z* = 8

(a) (6%) For elimination, find *l*21*, l*31, *l*32 and *E*21*, E*31*, E*32.

(b) (6%) Use *EA* = *U* to get *U* and solve for x.

(c) (6%) Use *A* = *LU*, solve *L*c = b and *U*x = c sequentially to get c and x.

6. (12%) Consider matrix *M* =

*Am×m Um×n Vn×m Dn×n*

, please answer the following questions:

(a) (4%) Please use block matrices *A*, *U*, *V* , *D* to find *M*2

(b) (4%) Suppose *A* is invertible and *Im*, *In* are *m × m* and *n × n* identity matrices. Please use block matrices *A*, *V* , *U*, *D* or thier inverses to find *Wm×m*, *Xn×m*, *Ym×n*, and *Zn×n* satisfying

*W* 0 *X In*

*A U V D*

=

*Im Y* 0 *Z*

*.*

*Im Y*

(c) (4%) Suppose *Z* is invertible. Find the inverse of

0 *Z*

*.*

*A U*

(d) (4%) Use the results of (a) and (b) to compute the inverse of

*V D*

.

7. (20%) Given a system S : *y* = *f*(*xa, xb, xc*), where *xa, xb, xc ∈* R are inputs and *y ∈* R is the output. By experiments, we get three samples whose inputs are (*xa, xb, xc*) = (1*,* 2*,* 3), (2*,* 1*,* 2) and (3*,* 2*,* 1), and output *y* = 1*,* 1 and 1 respectively. Try to answer following questions:

(a) (2%) To solve S, firstly we model S. The simplest one is to assume that *S* as a linear model, where *y* = *θaxa* +*θbxb* +*θcxc* with real-number parameters. Write down the equations for the three experiments in matrix form.

(b) (12%) According to (a), what are the elimination matrix *E* and decomposed matrices *L* and *U*?

(c) (2%) Please solve *θa, θb* and *θc*.

(d) (4%) Under what condition S does have no solution? Why? If we change *y* to 2*,* 1 and 3, is there solution to S under that condition?

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